

Sovereign Default and Capital Controls

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Abstract

This paper explores a role for capital control policies in enhancing a sovereign’s commitment to repay its debts. I study the equilibrium of an economy in which a sovereign must finance some expenditure, is constrained by the savings decisions of domestic households, and cannot discriminate between foreign and domestic lenders. I show that capital controls are crucial for implementing an equilibrium with lending from abroad when domestic disposable income is low. The distortion controls affect on bond prices crowds in domestic lending, enforcing repayment. In an environment with uncertainty over the cost of default, the sovereign exploits the commitment device that capital controls afford to mitigate, but not eliminate, default risk due to the distortionary cost of controls and the option value of default. The paper offers a novel rationale for countercyclical capital controls, distinct from conventional theories that stipulate controls be employed during expansions to regulate capital inflows. The optimal policy is characterized and empirical implications are discussed.

Keywords: Sovereign default, capital controls, optimal taxation

JEL Classification: F34, F38, H21

1. Introduction

During recessions a sovereign’s lack of commitment to repay its debts can lead to a sisyphean spiral in which a spike in bond yields leads to reductions in investment, further reductions in output, and, potentially, default. A sovereign in this environment would like to tie its hands in order to overcome the time inconsistency of its repayment decision. In this paper I explore capital controls as an instrument that can be employed to enforce this commitment to repay. In particular, this regulation of cross-border flows implements a wedge between the price of bonds faced by domestic and foreign lenders, making bonds relatively cheaper for domestic agents.

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This shift in the composition of lenders incentivizes a sovereign to repay when the ability to selectively default on foreign lenders is restricted.

In this paper, I show that capital controls are crucial in supporting lending from abroad when domestic disposable income is low, implying an optimal policy that is counter-cyclical. Capital controls act as a commitment device, implemented ex-ante to impose a wedge in the pricing equation for sovereign bonds. The wedge distorts the savings decisions of domestic agents and crowds out foreign lenders. This has the effect of mitigating default risk, allowing the constrained optimum to be achieved. The result offers a novel rationale for countercyclical capital controls, distinct from conventional theories.

In this context, I study a two period model populated by a benevolent sovereign that lacks commitment to repay, domestic households, and foreign lenders. The sovereign must finance some fiscal expenditure and has a motive to smooth the burden of this spending over time via the international capital market. It commits ex-ante to a capital control policy and a single bond issuance. These bonds are purchased by risk-neutral foreign lenders and domestic households privately optimizing their consumption and savings decisions. Default in the second period is non-discriminatory with respect to bondholder origin.

I show that in a decentralized environment, in which the decisions of sovereign and domestic households are distinct, capital controls are necessary to achieve the constrained optimal allocation of an economy. This is in contrast with the standard workhorse models of the literature (Arellano, 2008; Mendoza and Yue, 2012), in which the sovereign and domestic households are centralized and the two can be represented as a single agent.\(^2\) In a centralized economy, a benevolent planner constrained by its borrowing limit can optimally issue bonds to foreigners so that its incentive compatibility constraint on repayment binds. In the decentralized implementation, when undistorted household savings are insufficient, be it because of low domestic income or large government expenditures, then commitment to repay cannot be enforced. In order to overcome this, the sovereign regulates cross-border flows via capital controls, placing

\(^2\)Or lack of restrictions on lump-sum transfers allows the problem to be expressed as such.
a wedge between the rates of return on government debt experienced by domestic and foreign lenders. The wedge distorts domestic household savings upwards, crowds out foreign lending, and restores commitment to repay. This illustrates a duality between a quantity restriction in the centralized economy on the one hand, and a pricing restriction in the decentralized economy on the other.

The results offer a novel rationale for counter-cyclical capital controls in alleviating default risk, distinct from conventional theories. Further, the optimality of counter-cyclical controls is robust to the introduction of household labor/leisure choice and government labor taxes. With these additional fiscal tools, the sovereign must weigh a motive to simultaneously smooth labor distortions and relax the incentive compatibility constraint on repayment. In light of the former, the latter is executed more efficiently via capital controls.

I study equilibrium default by introducing uncertainty over the cost of failing to repay. In this environment optimal capital controls are utilized to mitigate, but not eliminate, default risk. The sovereign weighs the marginal benefit of increased bond prices due to more restrictive controls, against the the cost of distorting household savings decisions and decreasing the probability of being able to exploit the option value of future default. Since the foreign lenders that price debts are risk-neutral, the marginal increase in equilibrium bond prices due to increased capital controls is not sufficient to compensate the risk-averse government beyond some threshold. These results show that while these controls are likely to be an important tool in managing default risk in practice, they do not preclude equilibrium default.

This paper also contributes to the international economics literature on the empirical determinants of capital controls. I estimate panel data regressions and provide empirical evidence that government spending and sovereign spreads co-vary positively with capital inflow restrictions. This is in concert with the model’s prediction that controls should be tightened when domestic debt holdings are insufficient to ensure repayment, be it because of large debt stocks.

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3These include externalities that lead to over-borrowing (Korinek and Sandri, 2014) and price rigidities (Fahri and Werning, 2013). Neely (1999) provides an overview.

4See Rose and Eichengreen (2014) and Fernández et. al (2014)
or low domestic incomes.

This inverse relationship between debt-to-GDP ratios and financial openness is evident for a subset of Latin American countries (Mexico, Argentina, Ecuador, and Chile). Following eras of relative openness, these countries implemented controls in the face of the 1980’s Latin American debt crisis. In the period of international deregulation that followed these controls were relaxed. In the case of Argentina, controls were re-imposed surrounding the 2001 financial crisis and default. Figure 1 plots the financial openness measure due to Chinn and Ito (2006) against debt-to-GDP ratios for each country from 1970 to 2010.

![Figure 1: Openness and Debt](image)

1.1. Related Literature

This paper contributes to an extensive literature studying sovereign debt repayment decisions (Arellano, 2008; Mendoza and Yue, 2012) by considering the implications of divorcing the household and government problems and constraining the latter to resort to distortionary taxation.

This paper also contributes to a recent literature studying selective defaults. These papers generally either treat default across jurisdictions as joint (Malluci 2016; Perez 2016), or as dependent on creditor residence (Erce and Diaz-Cassou 2010; Erce 2012; Paczos and Shakhnov 2014).
Paczos and Shakhnov (2016), for example, consider an economy in which foreign and domestic agents hold distinct assets in their model, rendering default perfectly discriminatory. This branch of the literature belies the empirical evidence. Some authors have studied the effectiveness of debt repatriation in preventing default (Brutti and Sauré, 2014), and evidence of imperfectly selective defaults (Reinhart and Rogoff, 2011; Kohlscheen, 2009). Roubini and Setser (2004) document the difficulties associated with discriminating based on residence when both external and internal creditor hold identical instruments. This paper bridges these two literatures by studying joint default, while allowing the sovereign to engage in imperfect discrimination via capital controls.

An extensive literature considers capital controls and the notion that they can be employed to temper time inconsistencies (Fahri and Tirole, 2012), externalities that lead to over-borrowing (Korinek and Sandri, 2014) and price rigidities (Fahri and Werning, 2013). These models imply a use for controls in macro-prudential or exchange rate policy. In a recent paper, Schmitt-Grohé and Uribe (2017) study the over-borrowing channel and find a pro-cyclical capital control policy is optimal; driven by the Ramsey planner’s motive to avoid a binding collateral constraint. This paper offers a novel rationale for countercyclical capital controls distinct from these studies.

In a paper similar to this one, Mengus (2014) shows that limiting the precision of transfers in an environment with ex-post heterogenous agents and unobservable domestic portfolios produces endogenous costs of default since ex-post bailouts are inefficient. In his model, the use of capital controls to manipulate home bias in domestic portfolios and improve a sovereign’s ability to borrow is considered. He shows that controls are undesirable because of their impediment to international portfolio diversification. The present paper makes progress towards tightly characterizing the optimal control policy and studying its interaction with output, other fiscal tools, and equilibrium default decisions. Broner, Martin and Ventura (2010) consider capital controls as policy device for shutting down the repatriation channel in secondary markets ex-post. In contrast, this paper considers capital controls as a commitment device imposed to enhance repayment expectations.
2. The Model

In this section I first solve the canonical small open economy model without commitment to repay. I then proceed to illustrate how capital controls are crucial in the implementation of the optimal allocation in an economy in which the household and government problems are divorced. Capital controls drop out of the model naturally and have a clear interpretation.

Consider a two period \((t \in \{0, 1\})\) model, an endowment economy populated by risk-neutral foreign lenders and a benevolent sovereign. The sovereign must finance some expenditure \((g_0)\) and makes consumption/savings decisions on behalf of the domestic economy by choosing how much to borrow from abroad \((B_f)\), taking the price of bonds as given \((q)\). The sovereign lacks commitment to repay, and can, in period 1, default on its debts, in which case it suffers a cost to period 1 output \((\phi)\).

2.1. Sovereign

I introduce the problem of the sovereign recursively. In period 1 the sovereign decides whether to repay its debts, suffering exogenous penalty \(\phi\) in default. The repayment decision, \(\delta\), is
\[
\max_{\delta} \{\delta u(y_1 - B_f) + (1 - \delta)u(y_1 - \phi)\}.
\]
Since there is no uncertainty, foreign lenders fully anticipate period 1 default given the sovereign’s borrowing decision. Therefore the period 0 sovereign chooses between the autarchy and borrowing allocation. The government solves:
\[
\max \{V_{aut}, V_{rep}\} \text{ where}
\]
\[
V_{aut} = u(y_0 - g_0) + \beta u(y_1)
\]
And
\[
V_{rep} = \max_{c_0, c_1, B_f} \{u(c_0) + \beta u(c_1) \}
\]
\[
\text{st. } c_0 \leq y_0 - g_0 + qB_f
\]
\[
c_1 \leq y_1 - B_f, \quad B_f \leq \phi
\]
Where repayment is subject to the incentive compatibility constraint, \(B_f \leq \phi\). The usual conditions on utility are assumed - \(u' > 0, u'' < 0, \lim_{c \to 0} u'(c) = \infty\) and \(\lim_{c \to \infty} u'(c) = 0\).
2.2. Foreign Lenders

Foreign lenders are risk neutral and unconstrained. They discount the future at the risk-free rate \( R \). The break-even constraint for the pricing of sovereign bonds is therefore \( \frac{q}{\delta} - \frac{1}{R} = 0 \).

2.3. Equilibrium

An equilibrium in this economy is defined as follows:

**Definition 1.** An equilibrium is a price \( q \), sovereign policies \( (B_f, \delta) \), and allocations \( (c_0, c_1) \) such that

1. Given the policies of the sovereign, foreign lenders’ break even constraints are satisfied.
2. Given the period 0 sovereign’s policy, the sovereign’s period 1 repayment decision is optimal.
3. Subject to the period 1 sovereign’s policy and the bond pricing functional, the sovereign’s period 0 objective is solved.
4. Bond and goods markets clear.

2.4. Solution

From the foreign lender’s break even constraint prices are \( q = \frac{\delta}{R} \). Taking first order conditions yields

\[
u'(c_0) = \beta Ru'(c_1) + \mu R.
\] (1)

Where \( \mu \) represents the Lagrange multiplier on the incentive compatibility constraint \( (B_f \leq \phi) \). With respect to the full commitment case the sovereign borrows less from abroad when the incentive compatibility constraint binds in order to ensure repayment. At the constraint allocations are \( c_0 = y_0 + g_0 + \frac{\phi}{R} \) and \( c_1 = y_1 - \phi \) and \( \frac{\beta u'(c_1(\phi))}{u'(c_0(\phi))} < \frac{1}{R} \). The sovereign would like to borrow more, but due to lack of commitment it cannot. I refer to this as the centralized constrained optimal allocation.

3. Implementation and Capital Controls

In this section I show that capital controls are crucial for allowing the sovereign to implement the constrained optimal allocation in an environment where domestic households make consumption/savings decisions and the sovereign issues bonds and sets capital controls. The
decentralization illustrates a role for this policy instrument in environments where these decision makers are distinct.

Consider a similar two period endowment economy \((t \in \{0, 1\})\) populated by households making savings decisions, risk-neutral foreign lenders, and a benevolent sovereign. The sovereign must finance some expenditure \(g_0\), sets capital controls, and issues bonds \((B)\) that are purchased by domestic agents \((B_d)\) and foreign lenders \((B_f)\). Additionally, the government restricts foreign inflows ex-ante via capital controls, \(\tau\). In period 1 the government utilizes lump sum transfers \((T_1)\) to repay the entire stock of bonds \((B)\) or defaults and receives penalty \((\phi)\). Timing here is crucial - the capital control policy is implemented before bond markets open and, via its effect on the inter-temporal savings decision of domestic households, acts as a commitment device. I introduce the agents in turn, outlining the distinctions from the centralized economy of Section 2.

3.1. Households

Domestic households make consumption/savings decisions \((c_0, c_1, B_d)\) subject to the government’s policies and repayment expectations. Under repayment \((q > 0)\) the problem of the household is

\[
\max_{c_0, c_1, B_d} u(c_0) + \beta u(c_1)
\]

\[
\text{st.} \quad c_0 \leq y_0 - qB_d - T_0, \quad c_1 \leq y_1 + B_d - T_1
\]

Where \(B_d\) denotes household savings and \((T_0, T_1)\) denote the sovereign’s lump sum transfer scheme (described below). The household’s first order condition is

\[
q = \frac{\beta R u'(c_1)}{u'(c_0)}
\]

This represents the household implementability condition. A greater return on government debt, induced by a decrease in \(q\), increases household savings.

\[^5\text{In what follows I assume bonds are issued domestically. Capital controls that are interpreted as a tax on an investor of a particular dominion may be equivalently considered as a subsidy allotted to investors of the other.}\]
3.2. Foreign Lenders

As above, foreign lenders are risk neutral and discount the future at rate $\frac{1}{R}$. In this environment bonds are priced according to

$$q = \begin{cases} \frac{\delta}{R(1+r)} & \text{if } B_f > 0 \\ \frac{\beta u'(c_1)}{u'(c_0)} & \text{if } B_f = 0 \end{cases}$$

Where the pricing functional now includes the wedge induced by capital controls between the return on debt for foreign lenders and domestic households. In the case that there is no lending from abroad it is as if the domestic economy is in autarchy.

3.3. The Sovereign

Government budget constraints are

$$(1 - \delta)T_0 = g_0 - qB - q\tau B_f, \quad T_1 = \delta B + (1 - \delta)\phi$$

Where $(T_0, T_1)$ represent lump sum transfers, $\delta = 1$ denotes repayment, and $\phi$ represents the exogenous cost of default. In repayment I restrict the period 0 government’s ability to utilize lump sum transfers. This assumption is necessary to separate the problems of the sovereign and domestic households and is common in the literature on Ramsey taxation.

At time 1 the government makes its repayment decision

$$\max_{\delta} \{\delta u(y_1 - B_f) + (1 - \delta)u(y_1 - \phi)\}$$

Foreign lenders fully anticipate period 1 default given the sovereign’s total bond issuance and capital control policy. The period 0 sovereign chooses between the autarchy and borrowing allocations, the latter subject to the incentive compatibility constraint on repayment. It solves

$$\max \{V_{aut}, V_{rep}\}, \text{ where, as above}$$

$$V_{aut} = u(y_0 - g_0) + \beta u(y_1)$$ (2)

I utilize the primal approach, rewriting the sovereign’s problem under repayment in terms of
allocations.

\[ V_{rep} = \max_{c_0,c_1,B_f,B_d} u(c_0) + \beta u(c_1) \]

st. \[ c_0 \leq y_0 - g_0 + \frac{B_f}{R} \quad (3) \]

\[ u'(c_0)c_0 = u'(c_0)y_0 + \beta u'(c_1)B_d \quad (4) \]

\[ c_1 \leq y_1 - B_f \quad (5) \]

\[ B_f \leq \phi \]

Where, crucially, (5) collapses to the economy-wide constraint since period 1 expenditures are financed via lump-sum transfers.

### 3.4. Equilibrium

I define an equilibrium in this environment as follows:

**Definition 2.** An equilibrium is a price \((q)\), government policies \((\tau, B, \delta, T_0, T_1)\), and allocations \((c_0, c_1, B_d)\) such that

1. Given prices, government policies, and the repayment decision, households maximize.
2. Given the repayment decision and capital control policies, foreign lenders’ break even constraints are satisfied when they are the marginal investor.
3. Given period 0 sovereign policy decisions and the decisions of households, the sovereign’s period 1 repayment decision is optimal.
4. Subject to the time 1 sovereign’s policy, household savings decisions, and the bond pricing functional, the government budget constraint is satisfied and the sovereign’s period 0 objective is solved.
5. Bond and goods markets clear.

### 3.5. A Countercyclical Capital Control

In this richer set up, implementing the centralized constrained optimal allocation yields a natural role for capital controls. This instrument distorts bond prices directly, via the wedge it imposes in the bond pricing equation, and indirectly via its role in realigning repayment incentives. Taking first order conditions under repayment yields

\[ u'(c_0) = \beta Ru'(c_1) + \mu R. \]

As above, with respect to the full commitment case the sovereign curtails foreign borrowing when the incentive compatibility constraint binds in order to ensure repayment. From this follows the implementation result:
Proposition 1. If the incentive compatibility constraint on repayment is violated for \( \tau = 0 \) and 
\[
\frac{\beta Ru'(c_1(\phi))}{u'(c_0(\phi))} < 1
\]
then \( \tau > 0 \) at the optimum.

Put simply, the sovereign can implement the centralized constrained optimal allocation via a capital control policy. Crucially, without this instrument the market for its bonds would collapse. To see this, note that at price \( q = \frac{1}{R} \) foreign lending will violate the incentive compatibility constraint. Furthermore, a quantity control of the kind \( B_f = \phi \) will not satisfy the equilibrium definition, as domestic savings will be insufficient to satisfy the government budget constraint.

Implementation follows from the relationship between risk-neutral prices and the household implementability constraint, 
\[
q = \frac{1}{R(1+\tau)} = \frac{\beta u'(c_1)}{u'(c_0)},
\]
so the optimal policy is:
\[
\tau^* = \begin{cases} 
\frac{u'(c_0)}{\beta Ru'(c_1)} - 1 & \text{if } \mu > 0 \\
0 & \text{Otherwise}
\end{cases}
\]

At the constraint this policy is
\[
\tau^* = \frac{u'(y_0 - g_0 + \frac{\phi}{R})}{\beta Ru'(y_1 - \phi)} - 1
\]

This result implies an optimal capital control that is countercyclical. Controls are decreasing in initial domestic disposable income \((y_0 - g_0)\), and increasing in the relative discount rate \((\beta R)\). Crucially, the result does not depend on \(\beta R < 1\), but more generally \(\frac{\beta Ru'(c_1(\phi))}{u'(c_0(\phi))} < 1\). Capital controls are a natural bi-product in the implementation of the centralized constrained optimal allocations. I refer to this as the decentralized constrained optimal allocation.

When household savings are insufficient to satisfy the incentive compatibility constraint on repayment (due to low period 0 disposable income or high period 1 income) then it is optimal to impose capital controls. There is a duality between the quantity restriction on foreign lending in the centralized optimum \((B_f = \phi)\), and the pricing restriction imposed by capital controls decentralized optimum.
Numerical Solution & Further Characterization

I proceed to solve the decentralized economy numerically for a particular parameterization described in Table 1, and CRRA utility, \( u(c) = \frac{c^{1-\gamma}}{1-\gamma} \).

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>( \beta )</th>
<th>( R )</th>
<th>( \phi )</th>
<th>( y_0 )</th>
<th>( y_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.96</td>
<td>1.04</td>
<td>0.12</td>
<td>1.05</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Results are reported in Table 3 for the decentralized environment with full commitment (the first best), in the environment where the sovereign can implement capital controls (coinciding with the constrained optimum), and in the case where controls are not a policy instrument.

<table>
<thead>
<tr>
<th>Allocation</th>
<th>Environment</th>
<th>Welfare</th>
<th>Internal Total Debt</th>
<th>( \tau )</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Best</td>
<td>Commitment</td>
<td>1</td>
<td>0.5789</td>
<td>0</td>
</tr>
<tr>
<td>Constrained Optimum</td>
<td>Capital Controls</td>
<td>0.9985</td>
<td>0.7035</td>
<td>0.1666</td>
</tr>
<tr>
<td></td>
<td>No Commitment</td>
<td>0.9687</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Under the regime with capital controls the allocation coincides with the constrained optimum. Controls support foreign lending in equilibrium, alleviating the time consistency problem and the loss of utility experienced in the no commitment case. Welfare is reported ordinally, as calculated from the utilitarian value of the household’s consumption stream in each environment relative to the full commitment environment.

Figure 2 illustrates this graphically. Beyond some government spending threshold (\( g_0 \)) capital controls become necessary. The government’s capital inflow tax increases household savings, which crowd out foreign lending and establish an equilibrium with positive price. Likewise, controls are decreasing in time 0 output for a similar parameterization in which \( g_0 \) is held fixed. This illustrates the countercyclical character of the optimal capital control. Under this parameterization, an economy suffering a mild recession will smooth the loss to output via international capital markets. If the recession is particularly severe, capital inflows restrictions are imposed.

Proposition 2. For government spending obligations (\( g_0 \)) below the threshold \( \bar{g} \) defined by
$u'(y_0 - \bar{g} + \phi) = \beta Ru'(y_1 - \phi), \quad \tau = 0$ is optimal. For $g_0 > \bar{g}$ it is optimal to impose positive inflow controls ($\tau > 0$).

**Proof.** The proof follows immediately from Proposition 1. For $g_0 > \bar{g}$, it follows that $u'(y_0 - g_0 + \frac{\phi}{R}) > \beta Ru'(y_1 - \phi)$, yielding $\frac{\beta u'(c_1)}{u'(c_0)} < \frac{1}{R}$. Thus $\tau > 0$ is optimal.

The government budget constraint more clearly illustrates the role of domestic savings in this implementation. Define $\bar{B}_d$ as the undistorted household savings decision characterized by $\frac{\beta u'(c_1)}{u'(c_0)} = \frac{1}{R}$. Suppose the incentive compatibility constraint on repayment binds ($B_f = \phi$). There is some level of government expenditure $g_0 = \bar{g}$ for which $\bar{g} = \frac{B_d + \phi}{R}$ and $\tau = 0$ implements the constrained optimal allocation.

For $g_0 > \bar{g}$ the incentive compatibility constraint is violated for $\tau = 0$, as revenue is raised via additional foreign lending ($B_f > \phi$) to meet the government budget constraint, $g_0 = \frac{B_d + B_f}{R}$, this cannot be an equilibrium. From Proposition 1 it follows that $\tau > 0$ is the optimal policy.

4. **Introducing a Labor Tax**

In this section I relax the restriction on financing expenditures via bond sales and abstract from lump-sum taxation by allowing the sovereign to resort to distortionary labor taxation in each period. I first define the Ramsey problem in this environment and then proceed to
characterize the optimal policy. As above, I assume risk-neutral foreign lenders price bonds in repayment according to $q = \frac{1}{R}$. I define $\bar{V}$ as the value of default to the period 1 government.

4.1. Households

Households maximize consumption, face disutility of labor, and save in sovereign bonds. Income is linear in labor supply $n_t$. Utility is additively separable in consumption and labor and is given by $u(c_0) - v(n_0) + \beta[u(c_1) - v(n_1)]$. I assume the usual condition on labor disutility: $v : [0, 1] \to \mathbb{R}, v' > 0, v'' > 0$, along with the boundary conditions $v(0) = 0, v'(1) = \infty$. The problem of the domestic household is as follows:

$$\max_{c_0, c_1, n_0, n_1, B_d} \quad u(c_0) - v(n_0) + \beta[u(c_1) - v(n_1)]$$

$s.t.$

$$c_0 + qB_d \leq n_0(1 - \tau_{n_0})$$
$$c_1 \leq n_1(1 - \tau_{n_1}) + B_d$$
$$0 \leq n_t \leq 1 \forall t$$

In what follows I assume that the constraint on labor supply is not binding in equilibrium. This problem yields the following set of implementability conditions:

$$\frac{u'(c_0)}{\beta Ru'(c_1)} = (1 + \tau), \quad \frac{v_{n_0}}{u_{c_0}} = (1 - \tau_{n_0}), \quad \frac{v_{n_1}}{u_{c_1}} = (1 - \tau_{n_1})$$

4.2. The Sovereign

The sovereign is benevolent, must finance some expenditure $g_0$ at time 0, and faces budget constraints

$$g_0 = \tau_{n_0}n_0 + qB_d + (1 + \tau)qB_f, \quad \tau_{n_1}n_1 = B_d + B_f$$

The sovereign makes its default decision at $t = 1$, in which case it obtains utility $\bar{V}$. Time consistency of the period 0 sovereign’s tax policy requires $u(c_1) - v(n_1) \geq \bar{V}$. Feasibility requires $c_0 \leq n_0 - g_0 + \frac{B_f}{R}$ and $c_1 \leq n_1 - B_f$. 

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4.3. Equilibrium

An competitive equilibrium with repayment in this economy is defined as follows.

**Definition 3.** A competitive equilibrium with repayment is a price \((q)\), government policies \((\tau, B, \delta, \tau_{n_0}, \tau_{n_1})\), and a feasible allocation \((c_0, c_1, n_0, n_1, B_d)\) such that

1. Given prices and sovereign policies the allocation solves the household’s problem.
2. Given prices, sovereign policies, and the allocation, the government budget constraints are satisfied
3. Given time 0 sovereign policies, the time 1 sovereign’s incentive compatibility constraint on repayment is satisfied
4. Bond and goods markets clear.

4.4. Optimal Capital Controls

Following the Primal Approach, and eliminating taxes and prices from the household budget constraints yields

\[ c_0 u'(c_0) = n_0 v'(n_0) - \beta u'(c_1) B_d, \quad c_1 u'(c_1) = n_1 v'(n_1) + u'(c_1) B_d \]

For succinctness I define the following

\[ W(c_0, n_0) \equiv u(c_0) - v(n_0) + \lambda_0[n_0 v'(n_0) - c_0 u'(c_0)] \]
\[ W(c_1, n_1) \equiv u(c_1) - v(n_1) + \lambda_1[n_1 v'(n_1) - c_1 u'(c_1)] \]

The Lagrangian is therefore

\[ W(c_0, n_0) + \beta W(c_1, n_1) + \theta_0[n_0 + \frac{B_f}{R} - g_0 - c_0] + \theta_1[n_1 - B_f - c_1] \]
\[ + \lambda_0[-\beta u'(c_1) B_d] + \lambda_1[u'(c_1) B_d] + \mu \beta [u(c_1) - v(n_1) - \bar{V}] \]

Where the multiplier on the incentive compatibility constraint is defined by \(\mu \beta \equiv \mu\), and \(\theta_i\) and \(\lambda_i\) are the Lagrange multipliers on the resource constraints and household budget constraints, respectively. First order conditions are

\[ -\frac{W_{n_0}}{W_{c_0}} = 1, \quad -\frac{W_{n_1}}{W_{c_1}} = \frac{\theta_1 - \mu u'(n_1)}{\theta_1 - \mu u'(c_1)} \]
\[ W_{c_0} = \beta R(W_{c_1} + \mu u'(c_1)) \]
Combining with the household optimality conditions, and solving for labor taxes and the capital control yields the following expression.

\[
\tau^*_n = 1 + \frac{v'(n_0)}{u'(c_0)} W_{c_0}, \quad \tau^*_n = 1 + \frac{v'(n_1)}{u'(c_1)} W_{c_1} \frac{\theta_1 - \mu v'(c_1)}{\theta_1 - \mu u'(n_1)}, \quad \tau^* = \frac{u'(c_0) W_{c_1} + \mu u'(c_1)}{W_{c_0}} - 1
\]

In general, the optimal capital control will non-zero so long as \( \frac{u'(c_0)}{u'(c_1)} \neq \frac{W_{c_0}}{W_{c_1}} \). For further characterization I specify separable and isoelastic utility, \( u(c) - v(n) = c^{1-\gamma} - \alpha n^{1+\phi} \). This yields the following expressions for the optimal tax rates:

\[
\tau^*_n = 1 - \frac{1 - \lambda_0(1-\gamma)}{1 - \lambda_0(1+\varphi)}, \quad \tau^*_n = 1 - \frac{1 - \lambda_0(1-\gamma)}{1 - \lambda_0(1+\varphi)} \left( \frac{\theta_1 - \mu n_1^0}{\theta_1 - \mu c_1^0} \right) \), \quad \tau^* = \frac{\mu}{1 - \lambda_0(1-\gamma)}
\]

The following result follows:

**Proposition 3.** For separable and isoelastic utility in consumption and labor, when the incentive compatibility constraint on repayment is slack labor taxes are smoothed and capital controls are not imposed. When the constraint binds positive capital inflows are imposed and period 1 labor taxes are distorted upwards.

To see this result, note that when the incentive compatibility constraint is slack the conditions collapse to

\[
\tau^*_n = \tau^*_n = 1 - \frac{1 - \lambda_0(1-\gamma)}{1 - \lambda_0(1+\varphi)}; \quad \tau^* = 0
\]

This establishes the common labor tax smoothing and no capital taxes result. When the constraint binds the time 0 sovereign imposes capital inflow controls, \( \tau^* = \frac{\mu}{1 - \lambda_0(1-\gamma)} > 0 \) in order to ensure future repayment. The capital control increases the rate of return on government debt for domestic agents. This has the effect of increasing consumption in period 1 financed by increased domestic savings in period 0. This distortion alters the tax revenues required in period 1, thus altering \( \tau_{n_1} \) as stated in the proposition.

All else equal, the imposition of positive capital controls increases the stocks of bonds that must be repaid via distortionary taxation at \( t = 1 \). Further, we have that \( \tau_{n_1}^* > \tau_{n_0}^* \) so long
as $\alpha n_1^2 < c_1^{-\gamma}$. Crucially, the period 1 sovereign is compensated for the additional taxation of labor it must impose by decreased obligations to pay to abroad. In this way increased domestic debt holdings increases the value of repayment and capital controls enforce intertemporal commitment. Combining the optimality conditions for the case where the constraint binds

$$
\tau_{n_1}^* = 1 - \frac{\mu}{\tau^*(1 - \lambda_0 (1 + \varphi))} \left( \frac{\theta_1 - \mu \alpha n_1^\varphi}{\theta_1 - \mu c_1^{-\gamma}} \right)
$$

It is immediate that $\tau_{n_1}^*$ is an increasing function of $\tau^*$. The sovereign levies additional labor taxes with respect to an environment with commitment in order to offset the increased bond issuance necessitated by capital controls. This is because, for a given bond issuance, domestic debt holdings (government revenues) are unambiguously increasing (decreasing) in $\tau$.

As in the case without labor taxes, a binding incentive compatibility on repayment is associated with the imposition of capital controls, which enforce future commitment to repay. The desire to smooth labor taxes, in conjunction with the motive to efficiently relax the incentive compatibility constraint on repayment yields a role for this intertemporal commitment device.

5. Equilibrium Default

In this section I study equilibrium default by introducing uncertainty over the cost of default failing to repay. In equilibrium capital controls influence the distribution of bondholders, affecting expectations of repayment, and altering equilibrium bond prices. The sovereign internalizes this when it implements its capital control policy. These controls are thereby the channel by which the sovereign influences ex-post repayment incentives. Figure 3 summarizes this feedback loop.
5.1. Stochastic Default Cost

Let the cost of default, $\phi \sim F(\Phi)$ with positive support and probability density function $f(\phi)$. This uncertainty in the continuation value of repayment breaks the ex-ante certainty of meeting the incentive compatibility constraint that pervaded the above analysis. Define $\mathbb{E}[\delta] = 1 - P(\phi < B_f) = 1 - F(B_f(\tau))$.

The model proceeds as in Section 3. Prices are set by risk-neutral foreign lenders, $q = \frac{1-F(B_f(\tau))}{R(1+\tau)} \equiv \frac{\mathbb{E}[\delta]}{R(1+\tau)}$. I define the repayment set

$$Re(B_f) \equiv \{ \phi : u(y_1 - B_f) \geq u(y_1 - \phi) \}.$$

The household’s first order condition, $q = \frac{\beta \int_{\delta \in Re} u'(c_1) f(\phi) d\phi}{u'(c_0)}$, pins down domestic savings. Consumption allocations are

$$c_0 = u^{-1}(u(y_0 - qB_d)), \quad c_1^{rep} = u^{-1}\left(u\left(y_1 + \left(\frac{1}{1+\tau}\right)B_d - g_0R\right)\right),$$

$$\mathbb{E}[u(c_{1 \ \text{def}})] = \mathbb{E}[u(y_1 - \phi)].$$

I characterize the solution in terms of $\tau$, and then proceed to solve the model numerically. The problem of the time 0 government is

$$\max_{\tau} \ u(c_0) + \beta \left[ (1 - F(B_f)) u(c_1^{rep}) + F(B_f) \mathbb{E}[u(c_{1 \ \text{def}})] \right]$$

Where optimization is subject to the foreign lenders pricing schedule, the household’s problem described above, and the government budget constraint. The government’s first order condition pins down the optimal level of controls

$$u'(c_0) \zeta = \beta \left[ (1 - F(B_f)) u'(c_1^{rep}) \eta + \frac{\partial F}{\partial \tau} \left( \mathbb{E}[u(c_{1 \ \text{def}})] - u(c_1^{rep}) \right) \right].$$

Where $\frac{\partial F}{\partial \tau} = f(B_f(\tau)) \frac{\partial B_f}{\partial \tau}$ and
The first order condition contains the usual marginal utility of consumption terms, appropriately weighted, and an additional additive term describing the change in default probability due to a change in controls. The latter term is weighted by the difference in expected allocations in each contingency. This captures the sovereign’s internalization of the default contingency and the marginal benefit of additional controls.

The terms within $\zeta$ correspond to a direct pricing effect and a household savings effect. An increase in capital controls today increases bond prices through future repayment expectations via the *pricing* channel. Domestic savings are distorted upwards via the *savings* channel. The terms within $\eta$ correspond to a domestic household return effect corresponding to the repayment on prior savings, and a net revenue effect. More restrictive controls at time 0 restrict borrowing from abroad in equilibrium, meaning more resources are repaid domestically at time 1 via the *return* channel. The change in government revenue due to the imposition of controls means a different quantity of bonds need be repaid in the future via the *net revenue* channel. Controls both directly additively increase government revenue via the $\tau B_f$ term of the government budget constraint, and decrease government revenue via the change in prices.

The sovereign faces a fundamental tradeoff between taking advantage of low default costs tomorrow (via relaxed capital controls and more foreign lending) and bolstering bond prices today (via increased capital controls and less foreign lending). The way in which the risk-averse sovereign and risk-neutral foreign lenders comparatively evaluate risk creates a fundamental tension in this decision.

### 5.2. Numerical Solution

The above first order condition is a fixed point problem in $\tau$. Consider the case with log utility and $\phi \sim \ln \mathcal{N}(\mu_\phi, \sigma_\phi^2)$. I proceed to solve the model numerically using the parameterization described in Table 1, albeit for log utility, $g_0 = .3$, and a mean-preserving spread about $\mu_\phi =$
Figure 4 plots bond prices, and domestic and foreign holdings as functions of controls for high and low variances of $\phi$ across the $\tau$ space. The optimal capital controls are marked in each case. Note that the control is chosen to push prices towards the riskless limit and increase time 1 revenue.

![Figures showing domestic holdings, bond prices, domestic share, and foreign holdings as functions of capital controls for high and low variances.](image)

**Figure 4: Optimal Controls: The Effect of Uncertainty**

The pricing schedule and agents’ debt holdings are more sensitive to shifts in control policy for higher variances of the shock distribution. Domestic savings are a decreasing function of controls as prices are pushed up via the *pricing* channel. The *revenue* channel, however, means fewer bonds need to be issued in equilibrium to satisfy the sovereign’s financing requirement. The latter effect dominates, implying an increasing domestic debt share in $\tau$. With risk-free prices ($q = \frac{1}{R}$) the level of domestic bond holdings is increasing in the level of capital controls.

Figure 5 (left panel) illustrates the role of controls in mitigating default risk. Without a control policy the sovereign’s probability of repayment is approximately .3. The sovereign’s optimal choice of $\tau$ drastically increases this probability to approximately .875 but does not eliminate the default contingency all together. This is due to the combination of the distortionary costs.
associated with taxation and a decreasing marginal benefit of additional controls due to the option value of default.

Figure 5 (right panel), plots the optimal controls as a function of the variance of $\phi$. The limiting case, $\sigma^2_\phi = 0$, is equivalent to the certainty case characterized in Section 5. Optimal controls display an increasing and concave relationship in the variance of the default cost. This suggests an empirical prediction, in which sovereigns crowd in domestic lending when levels of uncertainty are high. Controls dull the pass-through of default risk onto debt prices, but do not mute it absolutely.

6. Empirical Evidence

In this section I evaluate the predictions of the model and provide empirical evidence that government spending and sovereign spreads co-vary positively with capital inflow restrictions. To do so I use the bond and overall inflow control indices of Fernández et. al (2016) as a measure of capital controls and estimate panel data regressions.

The relationship between spreads and inflow restrictions is particularly pronounced for a subset of emerging economies. Figure 10 plots mean 10-year bond spreads against the mean of the inflow restrictions index from 1995 to 2015. Captured in this period is both the late 1990s era of deregulation and the Great Recession-era policy reversals. This is in line with the
predictions of the model - that in a period of higher uncertainty, such as the latter, one would expect larger spreads and more prohibitive capital controls.

![Controls and Sovereign Risk](image)

Figure 6: Inflow Controls and Spreads: Emerging Economies

I estimate panel data regressions to assess the relationship between bond inflow restrictions, sovereign spreads, and government spending for a collection of small open economies. For robustness, both bond inflow restriction and overall inflow restriction indices are utilized. Country and country-time fixed effects are reported alongside the pooled results and GDP growth rates are included to control for business cycle effects. The results are tabulated in Table 3. As a robustness check I estimate the model using the more general overall inflow restriction index of Fernández et. al as well.

There is a significant and positive relationship between government spending and bond inflow restrictions when country or country-time fixed effects are included. This is in accordance with the model’s prediction that the imposition of controls is necessary when government spending needs are elevated and domestic savings capture alleviates default risk. The relationship between GDP growth rates and bond inflow controls is insignificant - substantiating previous findings that the business cycle is a weak determinant of these policies. The relationship between controls

---

6As in Aguiar and Gopinath (2007), I exclude all Group of Seven countries other than Canada to define a small open economy. The final sample is described in the accompanying Appendix A. The 10-year spread is defined as the difference between the rate on a sovereign’s 10-year bond and that of a 10-year U.S. Treasury.
Table 3: Panel Regressions

<table>
<thead>
<tr>
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<th>10-Yr Spread</th>
<th>Overall Inflow Restrictions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Bond Inflow Restrictions</td>
<td>0.0273***</td>
<td>0.0055**</td>
</tr>
<tr>
<td></td>
<td>(0.0091)</td>
<td>(0.0022)</td>
</tr>
<tr>
<td>Govt./GDP</td>
<td>-0.0155*</td>
<td>0.0155**</td>
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<tr>
<td></td>
<td>(0.0081)</td>
<td>(0.0075)</td>
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<tr>
<td>GDP growth</td>
<td>0.0239**</td>
<td>0.0023</td>
</tr>
<tr>
<td></td>
<td>(0.0109)</td>
<td>(0.0036)</td>
</tr>
<tr>
<td>Constant</td>
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</tr>
<tr>
<td></td>
<td>(0.1745)</td>
<td></td>
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<tr>
<td>Observations</td>
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<tr>
<td>R²</td>
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<td>0.0272</td>
</tr>
<tr>
<td>F Statistic</td>
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<td>5.32***</td>
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<td>Country FE</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Time FE</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

Note: *p<0.1; **p<0.05; ***p<0.01.

Estimation is via OLS. Clustered SEs are reported. Annual 10-yr spreads are calculated from a mean of monthly observations. 10-Yr Spreads are reported in percentage terms. “Country FE” are country dummy variables and “Time FE” are annual dummy variables.

and government spending does not extend to the more general inflow restriction index, indicating that policies restricting foreign participation in equity and money market activities, amongst others, are not implemented along the dimension of fiscal expenditure concerns.

Most notably, there is a robust and statistically significant positive relationship between 10-year sovereign bond spreads and the inflow restriction indices. These results, when combined with the mechanism explored above, provide a possible explanation for the observed pass-through from institutional development to controls found by Rose and Eichengreen (2014) and the persistence of control policies noted by Fernández et. al (2014). Lack of institutional commitment manifests itself in default risk, and the intertemporal commitment provided by capital controls suggests their implementation. These results are robust to alternative maturi-
ties and country income subgroup restrictions. The findings are in accordance with the model’s prediction that a higher ex-ante probability of default is contemporaneously associated with the imposition of inflow restrictions.

7. Concluding Remarks

In this paper I study a novel role for capital control policies in mitigating default risk when a sovereign cannot discriminate between foreign and domestic lenders. In the model these controls serve to support an equilibrium with foreign lending at the cost of distorting domestic savings. The empirical analysis provides suggestive evidence of a positive co-movement between control policies and sovereign risk.

The results offer an alternative rationale for the timing of capital control implementation, apart from conventional theories in which control schedules are driven by the business cycle or macro-prudential considerations. The interaction between these other policy motives alongside lack of commitment and default risk would be valuable avenue for further research.

References


