Online Appendix for Sovereign Default and Capital Controls

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1. Additional Proofs

1.1. Inflow tax/outflow subsidy equivalence

This section shows the equivalence between inflow controls levied upon foreign lenders when bonds are issued domestically and outflow subsidies levied upon domestic households when bonds are issued abroad.

1.1.1. Case 1: Tax on Inflows, τ

The break-even constraint of risk-neutral foreign lenders:

$$q(1+\tau) - \frac{1}{R} = 0 \tag{1}$$

$$q = \frac{1}{R(1+\tau)} \tag{2}$$

The government budget constraint in model without default:

$$g_0 - T_0 = qB + q\tau B_f \tag{3}$$

$$g_0 - T_0 = qB_d + qB_f + q\tau B_f \tag{4}$$

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Combining the foreign lender's condition (2) and the government budget constraint (4):

$$g_0 - T_0 = \frac{1}{R(1+\tau)} B_d + \frac{1}{R(1+\tau)} (1+\tau) B_f \tag{5}$$

$$g_0 - T_0 = \frac{1}{R(1+\tau)} B_d + \frac{1}{R} B_f \tag{6}$$

Note that all else equal the capital control appears to be revenue-reducing. However, the net revenue in equilibrium is a function of domestic household savings B_d as well. In the model with default, there will be another revenue-generating effect - namely the discontinuous effect of controls on the equilibrium repayment decision.

1.1.2. Case 2: Domestic subsidy, τ^s

When the control is a subsidy to domestic households, appearing in the household budget constraint as follows:

$$c_0 + q(1-\tau^s) B_d \leq y_0 - T_0 \tag{7}$$

Break-even constraint of foreign lenders in this case:

$$q - \frac{1}{R} = 0$$

$$q = \frac{1}{R}$$

Government budget constraint

$$g_0 - T_0 = qB - q\tau^s B_d$$

$$g_0 - T_0 = qB_d + qB_f - q\tau^s B_d$$

$$g_0 - T_0 = q(1-\tau^s) B_d + qB_f$$

where the subsidy is an outflow to domestic households. Combining yields:

$$g_0 - T_0 = \frac{1}{R}(1-\tau^s)B_d + \frac{1}{R}B_f$$

1.1.3. Equivalence

The subsidy to domestic households is equivalent to the tax on foreign lenders when

$$(1-\tau^s) = \frac{1}{(1+\tau)}$$

or

$$\tau = \frac{1}{(1-\tau^s)} - 1$$

For intuition, note that in the case of a subsidy the effective price domestic households pay for a bond is $q(1-\tau^s) = \frac{(1-\tau^s)}{R}$, whereas with a tax on foreigners this price is $q = \frac{1}{R(1+\tau)}$. That is to say, one can equivalently represent a sovereign's bond issued **domestically** with a *tax* on foreign inflows and a bond issued **abroad** with a *subsidy* on domestic outflows. The underlying assumption in both cases is that the sovereign's debt is the only asset domestic households can purchase.

2. Model Extensions

2.1. Restrictions on T_0

The baseline decentralized economy restricts the sovereign's period 0 lump-transfers to be set to zero unless the prevailing equilibrium price is zero, as follows:

$$g_0 = \begin{cases} qB + q\tau B_f \text{ if } q > 0 \\ T_0 \text{ otherwise} \end{cases} \qquad T_1 = \delta B + (1-\delta)\phi \tag{8}$$

Relaxing this assumption, and allowing the sovereign to use this policy independent of the prevailing market price of debt, the government budget constraint is as follows:

$$g_0 = qB_d + qB_f + q\tau B_f + T_0 \tag{9}$$

Taking the problem of the household in the decentralized economy

$$\max_{c_0, c_1, B_d} u(c_0) + \beta u(c_1) \tag{10}$$

$$\text{st.} \quad c_0 \leq y_0 - qB_d - T_0, \quad c_1 \leq y_1 + B_d - T_1 \tag{11}$$

Without the restriction on T_0 the period 0 household budget constraint and government budget constraint can be combined

$$c_0 \leq y_0 - qB_d - g_0 + qB_d + qB_f + q\tau B_f \tag{12}$$

$$c_0 \leq y_0 - g_0 + q(1+\tau)B_f \tag{13}$$

$$c_0 \leq y_0 - g_0 + \frac{B_f}{R} \tag{14}$$

Thereby obtaining the constraint in the centralized Eaton/Gersovitz economy (Equation 3 in the paper).

2.2. Income fluctuations

In this section I consider an extension of the model in which there is uncertainty over period 1 income. I adopt the functional form of Arellano (2008). When the government defaults income is $h(y_1) < y_1$ where

$$h(y_1) = \begin{cases} y_1 - \phi & \text{if } y_1 > \hat{y} \\ y_1 & \text{if } y_1 <= \hat{y} \end{cases} \tag{15}$$

For simplicity, I solve the model for the case in which $y_1 \in \{y_1^{low}, y_1^{high}\}$, occurring with probabilities $1 - p$ and p, respectively. Assume $\hat{y} = y_1^{low}$, so that there is no cost of defaulting when the economy is in the low income state. The household Euler equation now holds in expectation

$$q = \frac{\beta\Big[(1-p)*u'(c_1^{low}) + (p)*u'(c_1^{high})\Big]}{u'(c_0)} \tag{16}$$

Let $\{\delta^{low}, \delta^{high}\}$ denote equilibrium repayment decisions in each income state. With no cost of default in the low income state the government will always default when y_1^{low} is realized because the autarky allocation is strictly preferred to repayment: $u(c_1^{def,low}) = u(y_1^{low}) > u(y_1^{low} - B_f) = u'(c_1^{rep,low})$ for any $B_f > 0$.

The break-even constraint of risk-neutral foreign lenders is:

$$\frac{q(1+\tau)}{(1-p)\cdot\delta^{low} + p\cdot\delta^{high}} - \frac{1}{R} = 0 \tag{17}$$

Re-ordering to obtain prices under foreign lending

$$q = \frac{p\cdot\delta^{high}}{R(1+\tau)} \tag{18}$$

As above, capital controls have a discontinuous effect on default decisions. Note $\delta^{high} = \mathbb{I}_{B_f \leq \phi}$. In the case that the incentive compatibility constraint on on repayment ($B_f \leq \phi$) is violated, repayment is restored by a policy

$$\tau^* = \frac{u'(y_0 - g_0 + \frac{p}{R}\phi)}{\beta R\big[p\cdot u'(y_1^{high} - \phi) + (1-p)\cdot u'(y^{low})\big]} - 1 \tag{19}$$

Some comparative statics to note

- **Repayment probability,** p: the optimal capital control is decreasing in the probability of the high state.
  - From (12), the probability on the y_1^{high} state has a first order effect. As the probability of the high income realization increases, domestic savings decrease, and a higher τ is

needed to stimulate domestic savings.

– There is an offsetting second order effect of increased p due to a decline period 0 marginal utility. This is due to the increased revenue generated from bond sales to foreigners as bond prices rise.

- **Income dispersion,** $(y_1^{high} - y_1^{low})$: For a mean-preserving spread of y_1 about y_0, increased dispersion is related to a decrease in the optimal capital control. Risk-averse domestic households increase savings due to increased income risk, offsetting the need for controls.

2.3. An Alternative Formulation

This section presents an alternative solution of the baseline model presented in the main paper. Writing the Ramsey problem in terms of domestic allocations and substituting in the government budget constrain, provides some additional intuition. This renders the economy-wide budget constraints redundant. The sovereign's problem under this equivalent formulation is

$$V_{rep}^{B_d} = \max_{c_0, c_1, B_d, B_f} u(c_0) + \beta u(c_1)$$

$$st.$$

$$u'(c_0)c_0 \leq u'(c_0)y_0 - \beta u'(c_1)B_d \tag{20}$$

$$c_0 \leq y_0 - g_0 + \frac{B_f}{R} \tag{21}$$

$$u'(c_0)c_1 \leq u'(c_0)y_1 - R\Big(u'(c_0)g_0 - B_d\beta u'(c_1)\Big) \tag{22}$$

$$B_d \geq \frac{u'(c_0)}{\beta R u'(c_1)}(g_0 R - \phi) \tag{23}$$

Where (6) and (8) describe the household budget constraints, (7) is the economy-wide resource constraint, and (9) has the natural interpretation of a constraint on domestic bond market participation. If domestic bond purchases are insufficient, the autarchic allocation is realized.

The constraint (6) clarifies the complementarity between domestic savings and repayment. An increase in domestic savings (B_d) decreases consumption, but also has the second order effect of increasing the return to savings($\frac{\beta u'(c_1)}{u'(c_0)}$). However, the increased rate of return allotted to domestic households, means additional bond issuance is required to to finance government expenditures, tightening the participation constraint (9). These tradeoffs will be fundamental with the introduction of uncertainty below.

Assigning Lagrange multipliers ψ, θ, μ to constraints (6), (8), and (9), respectively, and combining the resulting expressions yields

$$\beta R u_{c_1} - u_{c_0} = \frac{\mu}{\theta}\left[u_{cc_0}\left(\frac{\beta u_{c_1}}{u_{c_0}}B_d + \frac{\phi}{R} - g_0\right) + u_{cc_1}\left(\frac{u_{c_0}}{u_{c_1}}B_d + \frac{u_{c_0}^2}{\beta R u_{c_1}^2}(\phi - g_0 R)\right) - u_{c_0}\right] \quad (24)$$

If the domestic participation constraint does not bind ($\mu = 0$) then the undistorted Euler equation is recovered. For the case that it does ($\mu > 0$), notice that the u_{cc_0} term is simply the government budget constraint restated at $B_f = \phi$. Similarly, the u_{cc_1} term is simply the participation domestic constraint (9), which binds in this case. Therefore we can express the condition for optimality

$$u'(c_0) = \beta R u'(c_1)\left(\frac{\theta}{\theta - \mu}\right) \quad (25)$$

This implies that $u'(c_0) > \beta R u'(c_1)$. From the household's first order condition we have that $u'(c_0) = \beta R u'(c_1)(1+\tau)$. Thus $\tau > 0$ at the optimum. This allocation ($B_f = \phi$), again coincides with the solution in the constrained planner's problem. When household savings are insufficient to satisfy the incentive compatibility constraint on repayment (due to low period 0 disposable income or high period 1 income) then it is optimal to impose capital controls.

There is a duality between the quantity restriction on foreign lending in the constrained planner's problem ($B_f = \phi$), and the pricing restriction imposed by capital controls in the implementation.

2.4. Anonymous Markets

The results of the above sections rely on the sovereign's ability to fully observe the aggregate distribution of bondholdings across domiciles. When bond markets are anonymous and households have some private information that governs their savings decisions this distribution is directly unobservable to the sovereign. Capital controls, however, act to overcome this information friction.

In this environment non-zero capital controls break the anonymity of bond markets and, in doing so, provide information that can be exploited by the period 1 sovereign. Market participants internalize this, causing the sovereign bond market to collapse under a regime with capital restrictions. I first describe the market and information structures of the economy and characterize the optimal policy generally, before proceeding to solve a simple case with linear utility that illustrates the mechanism clearly.

2.4.1. Market Structure

The model follows largely from the endowment economy of Section 3, albeit with distinct assumptions on the structure of information. I assume the representative domestic household has some private information regarding its liquidity preference $\beta^i \in \{\beta^L, \beta^H\}$. Further, bond markets are anonymous, rendering the savings decisions of domestic households unobservable to the sovereign. For simplicity I assume that bond market participants (domestic households, foreign lenders) are able to identify one another.

Ex-ante the sovereign has beliefs μ over domestic liquidity preferences β^i, and thereby domestic savings B_d^i, assessing probability π^i, $i \in \{L, H\}$, to each state. Define $\pi = \pi^L$, $1-\pi = \pi^H$. The government is only able to impute the distribution of bondholders across domiciles ex-post from the components of its budget constraint

$$(1-\delta)T_0 = g_0 - qB - q\tau B_f$$

This provides the fundamental tradeoff presented by capital controls. The information set of the sovereign at time 0 is

$$\mathcal{F}_0 = \{y_0, y_1, g_0, \mu\}$$

While the time 1 information set depends on time 0 policies

$$\mathcal{F}_1 = \begin{cases} \{y_0, y_1, g_0, q, B, \mu\} & \text{if } \tau = 0 \\ \{y_0, y_1, g_0, q, B_d, B_f\} & \text{otherwise} \end{cases}$$

Where $F_1^{\tau=0} \subseteq F_1^{\tau\neq 0}$, so that capital controls serve to augment the information available to the period 1 government. The sovereign cannot commit to avoid exploiting this information at the time of its repayment decision.

2.4.2. Optimal Policies

From Section 3, under full information, the optimal policy for each state $i \in \{L, H\}$ is

$$\tau^i = \begin{cases} \frac{u'(c_0)}{\beta^i R u'(c_1)} - 1 & \text{if } \mu > 0 \\ 0 & \text{Otherwise} \end{cases}$$

Meaning that the capital control policy that supports lending in both states, when the incentive compatibility constraint binds, is

$$\tau^* = \max_{i \in \{L,H\}} \tau^i = \tau^L \tag{26}$$

This policy introduces additional distortions in the β^H state in order to support lending in the β^L in which domestic agents are relatively more impatient. However, it is not necessarily the case that this policy is optimal in the environment where bond markets are anonymous. Positive capital controls reduce the information set of the period 1 sovereign to a singleton, inducing the incentive compatibility constraints on repayment

$$u(y_1 - B_f^i) \geq u(y_1 - \phi) \text{ for each } i$$

where B_f^i indicates the borrowing from abroad in each state. The ex-ante expected payoff of the policy in (13) is

$$V_{\tau^*}^{rep} = \pi^L \left[u\left(y_0 - g_0 + \frac{\phi}{R} \right) + \beta^L u(y_1 - \phi) \right] + \pi^H \left[u\left(y_0 - g_0 + \frac{B_f^H}{R} \right) + \beta^H u(y_1 - B_f^H) \right]$$

In the case that borrowing is constrained in both states, $B_f^H < \phi$. Capital controls have the effect of supporting borrowing from abroad across both states at the expense of constricting borrowing from abroad in the $i = H$ state beyond that of the observable markets case. On the other hand a lack of capital restrictions, because it does not reveal the aggregate distribution of bond holdings across domiciles, induces the constraint

$$\sum_{i \in \{L,H\}} \pi^i u(y_1 - B_f^i) \geq u(y_1 - \phi)$$

Yielding ex-ante expected payoff

$$V_{\tau=0}^{rep} = u\left(y_0 - g_0 + \frac{E[B_f^i]}{R} \right) + (\pi^H \beta^H + \pi^L \beta^L) u(y_1 - E[B_f^i])$$

The sovereign's decision to implement capital controls in this environment hinges on whether this commitment device supports a better ex-ante expected payoff than the commitment to maintain bond market anonymity.

Capital controls support an allocation by distorting domestic savings directly, while beliefs serve to support an allocation by relaxing borrowing constraints in some states at the expense of others. To illustrate this point and further characterize the solution, I solve a simple case under risk-neutral household utility.

2.4.3. A Risk-neutral Case

Define the foreign lender's discount factor $\beta^F \equiv \frac{1}{R}$. Assume $\beta^i = \beta^H = \beta^F$ with probability π and $\beta^i = \beta^L < \beta^F$ with probability $1 - \pi$. The period 1 sovereign is uncertain of the identity of its bondholders when the repayment decision is made unless it reveals this information is revealed via the policies of the period 0 sovereign. I assume foreign lenders can observe whether or not domestic agents are participating in bond markets.

Households are atomistic, risk neutral and maximize consumption according to

$$
\begin{aligned}
V = \max_{c_0,\, c_1,\, B_d} \; & c_0 + \beta^i[c_1] \\
& st. \\
& c_0 = y_0 - qB_d - T_0 \\
& c_1 = y_1 + \delta B_d - T_1 \\
& c_t > 0 \; \forall t \\
& B_d \geq 0
\end{aligned}
$$

Assume $y_0 > g_0 > \phi$ so that sufficient domestic resources exist to finance government expenditures at time 0 and default can occur in equilibrium. Suppose the period 1 sovereign could perfectly observe β^i. In the β^L state borrowing is solely from abroad, and default is optimal. Allocations are

$$
\begin{aligned}
c_0 &= y_0 \\
c_1^{rep} &= y_1 - \frac{g_0}{\beta^F} \\
c_1^{def} &= y_1 - \phi \\
c_1^{rep} &< c_1^{def}
\end{aligned}
$$

Thus in the β^L state the international market for debt collapses, and the autarchic allocation results.

$$
V^{aut} = y_0 - g_0 + \beta^L[y_1]
$$

2.4.4. Capital Controls to Support Markets

Under full information the market for debt collapses in the β^L case. When inflow controls are imposed borrowing from abroad can be supported. Prices are determined by

$$q = \frac{\beta^F \delta}{(1+\tau)}$$

Suppose the sovereign carefully sets $(1+\tau) = \frac{\beta^F}{\beta^L}$. External lenders now offer prices $\beta^L \delta$. Consumption allocations are

$$\begin{aligned} c_0 &= y_0 - \beta^L B_d \\ c_1^{rep} &= y_1 + \frac{\beta^L}{\beta^F} B_d - \frac{g_0}{\beta^F} \\ c_1^{def} &= y_1 - \phi \end{aligned}$$

Repayment is optimal so long as

$$B_d > \frac{\beta^F}{\beta^L}\left(\frac{g_0}{\beta^F} - \phi\right) \tag{27}$$

The implementation of capital controls permits borrowing from abroad and allows front-loading consumption.

2.4.5. Capital Controls to Kill Markets

In an environment with period 0 bond market anonymity, the sovereign can do better by adhering from implementing capital inflow restrictions and maintaining this anonymity. The default decision in all cases amounts to comparing expected consumption allocations under each policy.

$$\begin{aligned} \mathbb{E}[c_1^{rep}] &= y_1 + \mathbb{E}[B_d] - B = y_1 - \mathbb{E}[B_f] \\ c_1^{def} &= y_1 - \phi \end{aligned}$$

This system of beliefs can support the following allocations

$$V^{\beta^L} = y_0 + \beta^L \left[y_1 - \frac{g_0}{\beta^H} \right]$$
$$V^{\beta^H} = y_0 - \beta^H B_d + \beta^H \left[y_1 - \frac{1}{\pi} \left(\phi - (1-\pi) \frac{g_0}{\beta^H} \right) \right]$$

Compared to those with capital controls imposed

$$V_{\tau^*}^{\beta^L} = y_0 - \beta^L B_d + \beta^L \left[y_1 + \frac{\beta^L}{\beta^H} B_d - \frac{g_0}{\beta^H} \right]$$
$$V_{\tau^*}^{\beta^H} = y_0 - g_0 + \beta^H [y_1]$$

Table 3 compares the borrowing from abroad under each control policy and realization of β^i.

Table 1: Controls and Uncertainty

$1+\tau$	β^L	β^H
0	$B_f = \frac{g_0}{\beta^H}$	$B_f \leq \frac{1}{\pi}[\phi - \frac{g_0}{\beta^H}(1-\pi)]$
$\frac{\beta^L}{\beta^H}$	$B_f \leq \phi$	$B_f = 0$

Consider the case without uncertainty. Since it is always optimal to borrow from the most patient agent, the optimal capital control policy is to tax foreign purchases ($1+\tau = \frac{\beta^F}{\beta^L}$) in order to obtain domestic participation. These subsidies will preclude foreign borrowing when $\beta^i = \beta^H$ is observed, but because the sovereign is indifferent between borrowing from each agent in this state the implementation of this policy is a Pareto improvement.

When identification is not assumed the sovereign can credibly commit to not reveal the identity of its bond holders by setting $\tau = 0$. Under a regime in which $\tau \neq 0$ the sovereign reveals the distribution of bondholders via the components of its budget constraint. Thus the constraints in the $1 + \tau = \frac{\beta^F}{\beta^L}$ case are identical to the full identification case.

When bond market anonymity is maintained the sovereign can support lending from abroad in the β^L state at the expense of tightening its borrowing constraint in the β^H state. There is no trade-off here because of the sovereign's indifference across lenders in this state. The unambiguous objective of the sovereign is to relax the borrowing constraint it faces in the β^L state. To this end it is optimal to commit to not identify its bondholders and to implement a control policy $\tau = 0$. Here this achieves the first best (full commitment) allocation under the specified beliefs.

While simple, this example is illustrative of the trade-off presented by capital controls in an environment in which markets are anonymous and bondholder identity can be obscured. Uncertainty allows beliefs to support relaxed borrowing constraints in some states at the expense of others. Capital controls, on the other hand, restore certainty and directly distort domestic savings incentives across states thereby restoring repayment.

3. Data Appendix

The inflow control indices of Fernández et. al (2016) are utilized as a measure of capital controls. The overall inflow index extends from 1995 to 2016, while the bond inflow index begins in 1997.

Government spending statistics are collected from World Bank National Accounts and OECD National Accounts datasets. 10-year bond yield data come from the *investing.com* World Government Bond database, FRED, and respective central bank databases. The 10-year spread is defined as the difference between the rate on a sovereign's 10-year bond and that of a 10-year Treasury. Monthly yields are annualized via a simple arithmetic mean.

GDP growth rates from the IMF World Economic Outlook are included to control for business cycle effects. Any country with less than 10 years of joint observations was dropped from the data set. As in Aguiar and Gopinath (2007), I exclude all Group of Seven countries other than Canada to define small open economies and to obtain the final sample.

The final collection of small open economies includes Australia, Austria, Belgium, Bulgaria, Canada, Chile, China, Colombia, Czech Republic, Denmark, Finland, Ghana, Greece, Hong Kong, Hungary, Iceland, India, Indonesia, Ireland, Israel, Kenya, Korea, Malaysia, Malta, Mexico, Netherlands, New Zealand, Norway, Pakistan, Philippines, Poland, Portugal, Romania, Russia, Singapore, Slovenia, South Africa, Spain, Sweden, Switzerland, Thailand, Turkey, and

Vietnam.